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This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should know that "quotient" indicates division and be able to: (1) divide a fraction of the form a/b , where 0 is less than (ab) and these are less than 100, by a positive integer less than 100, (2) divide a fraction of the form a/b by a fraction of the form c/d , where 0 is less than (ab, c, d) and these are less than 100, (3) divide mixed numbers by mixed numbers of the form Xa/b , where 0 is less than (Xab) and these are less than 100, (4) divide literal fractions, and (5) divide any combination of the letters, fractions, integers, and mixed numbers listed above. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

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FINAL REPORT
Project No. OE7-003i
Contract No. OEG-4-7-070031-1626
Report No. 16-H

Occupational Mathematics
DIVISION OF FRACTIONS

June 1968

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U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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Occupational Mathematics

DIVISION OF FRACTIONS.

Project No. OE7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-H

by
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June 1968

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Washington State University, Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

OBJECTIVES

1. The student should know that the word "quotient" indicates the operation of division.
2. The student should be able to divide a fraction of the form a/b , where $0 < (a,b) < 100$, by a positive integer less than 100.
3. The student should be able to divide a fraction of the form a/b by a fraction of the form c/d , where $0 < (a,b,c,d) < 100$.
4. The student should be able to divide mixed numbers by mixed numbers where the mixed numbers are of the form $X a/b$ and $0 < (x,a,b) < 100$.
5. The student should be able to divide literal fractions.
6. The student should be able to divide any combination of the letters, fractions, integers, and mixed numbers listed above.

Page B

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

- | | |
|----------|---------------------|
| (a) Yes | Turn to page 1 |
| (b) No | Turn to page C |
| (c) HELP | Go see your teacher |

Page C

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it,
you will probably be ready to begin.

Hello. Do you have trouble finding the quotient of two fractions? This unit on division of fractions is for you.

Let's begin by asking a few simple questions concerning the division of fractions.

What is the quotient when we divide 12 by 4?

(a) 12

Turn to page 5

(b) 4

Turn to page 2

(c) 3

Turn to page 3

Incorrect.

In the problem of 12 divided by 4, the answer is 3.
Now 12 is called the dividend, the 4 is called the
divisor, and the answer is called the quotient.

Example: $10 \div 5 = 2$, and 2 is the quotient.

Another: $12/3 = 4$; and, in this case, the 4 is the
quotient.

What is the quotient of 14 divided by 7?

(a) 7

Turn to page 4

(b) 2

Turn to page 3

That's correct!

Now continue.

What is the quotient of $2/3 \div 5/7$?

(a) $14/15$

Turn to page 18

(b) $10/21$

Turn to page 11

(c) Help

Turn to page 15

No.

7 is the divisor and 14 is the dividend.

Go back to page 2 and try again by reading the preceding information carefully before answering the question.

Incorrect.

In the problem of 12 divided by 4, the answer is 3.
Now 12 is called the dividend, the 4 is called the divisor, and the answer is called the quotient.

Example: $10 \div 5 = 2$, and 2 is the quotient

Another: $12/3 = 4$; and, in this case, the 4 is the quotient.

What is the quotient of 14 divided by 7?

(a) 7

Turn to page 4

(b) 2

Turn to page 3

So far so good! Now...

The only thing you can multiply a fraction by and not change its value is 1 (one).

Therefore,

$$\frac{2/3}{1/2} = \frac{2/3}{1/2} \times 1 = \frac{2/3}{1/2} \times \frac{\bigcirc}{2/1} .$$

What number goes in the \bigcirc ?

(a) $1/2$

Turn to page 15

(b) $2/3$

Turn to page 7

(c) 2

Turn to page 10

Wrong!

$\frac{2/3}{1/2}$ doesn't equal one (1).

Go back to page 6 and try again.

4/5 is the correct answer.

$$5/8 \div 5/2 = ?$$

(a) 4

Turn to page 75

(b) 1/4

Turn to page 18

(c) Help

Turn to page 15

(d) 25/16

Turn to page 9

Page 9

You multiplied instead of dividing!

Go back to page 8 and try again.

Correct again!

Now, since $\frac{2/3}{1/2} \times \frac{2}{2} = \frac{2/3}{1/2}$, we get

$$\frac{2/3}{1/2} = \frac{2/3}{1/2} \times \frac{2}{2} = \frac{2/3 \times 2}{1/2 \times 2} = ?$$

- (a) $\frac{4/3}{1}$ Turn to page 22
- (b) $\frac{4/3}{1/4}$ Turn to page 28
- (c) $1/3$ Turn to page 19

Incorrect.

Let's see how the problem was worked.

$$\frac{2}{3} \div \frac{5}{7} = \frac{\frac{2}{3}}{\frac{5}{7}} = \frac{\frac{2}{3} \times \frac{7}{5}}{\frac{5}{7} \times \frac{7}{5}} = \frac{\frac{2}{3} \times \frac{2}{5}}{1} =$$

$$\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}.$$

Notice that division of fractions is just reducing a complex fraction.

Let's see if you can work one now.

$$\frac{3}{5} \div \frac{2}{3} = ?$$

(a) $\frac{10}{9}$

Turn to page 16

(b) $\frac{9}{10}$

Turn to page 13

(c) Help

Turn to page 15

Good, $2/3 \times 1/5$ is correct!

So, $2/3 \div 5 = 2/3 \times 1/5 = ?$

- | | |
|------------|-----------------|
| (a) $10/3$ | Turn to page 24 |
| (b) $2/15$ | Turn to page 50 |
| (c) $2/8$ | Turn to page 34 |

That is correct!

Let's do another problem.

$$\frac{2}{3} \div \frac{5}{6} = ?$$

(a) $\frac{4}{5}$

Turn to page 8

(b) $\frac{5}{4}$

Turn to page 16

(c) $\frac{5}{9}$

Turn to page 15

Incorrect.

Dividing by 2 is the same as dividing by the fraction $\frac{2}{1}$. Hence,

$$\frac{4}{7} \div 2 = \frac{4}{7} \div \frac{2}{1} = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14} = \frac{2}{7}.$$

Find $\frac{2}{3} \div 3$.

(a) 2

Turn to page 15

(b) $\frac{2}{9}$

Turn to page 50

A Procedure for Division of Fractions

As a link to the unit on multiplication, let us consider the model for how many objects there would be in $\frac{1}{4}$ of a set of 12 objects. This could be done by taking the 12 objects and dividing the set into 4 equal parts. (See figure at the right.)

From this, it is plain to see that $\frac{1}{4}$ of 12 is 3.



However, we should also note that this model illustrates $12 \div 4$ as well.

[model for $\frac{1}{4} \times 12$
and for $12 \div 4$]

Continued on next page

Question:

Which model shows $\frac{1}{5}$ of 10?

(a)



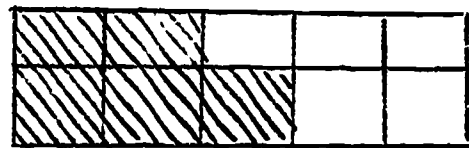
(b)



(c)



(d)



Answers:

(a) Turn to page 25

(b) Turn to page 30

(c) Turn to page 21

(d) Turn to page 30

Nope. Wrong one.

Let's take a closer look at division.

$2/3 \div 1/2 =$ the fraction:

(a) $\frac{2/3}{1/2}$

Turn to page 17

(b) $\frac{1/2}{2/3}$

Turn to page 15

Good! $2/3 \div 1/2$ equals the fraction $\frac{2/3}{1/2}$.

Now, how can we made the denominator become one (1)?

- | | |
|--|------------------|
| (a) Help | Turn to page 15 |
| (b) Just change the 2 to 1 | Turn to page 15 |
| (c) Multiply by its reciprocal | Turn to page 6 |
| (d) I don't know what a reciprocal
is | Turn to page 100 |

That's correct. You're doing very well!

Now, what is $9/10 \div 3/5$?

(a) $2/3$

Turn to page 27

(b) $3/2$

Turn to page 40

(c) $45/30$

Turn to page 38

You didn't multiply correctly.

What happened?

(a) I made a careless mistake

Turn to page
10 and try again

(b) I need a little help with
multiplication of fractions

Go to Unit 6 and
review multiplication
of fractions

Wrong.

Division of any number by one leaves it unchanged.

Examples: $5 \div 1 = 5$

$$21 \frac{1}{2} \div 1 = 21 \frac{1}{2}$$

$$\frac{2}{5} \div 1 = \frac{2}{5}$$

So what is $\frac{4}{3}$ divided by one?

Go back to page 22 and do the problem again.

I'm afraid you don't understand.

You should:

- (a) Go ask your teacher for help and then come back to page 15

OR

- (b) Go back and reread page 15 and make another choice.

Very good!

Now, what is $\frac{4/3}{1}$ equal to as a simple fraction?

(a) $4/3$

Turn to page 13

(b) $3/4$

Turn to page 20

Now Really!

This model



shows 5 equal sets of 5, which has a total of 25 objects. Therefore, this model illustrates 25 divided by 5.

Go back to page 25 and make another selection.

Incorrect.

Surely you remember that to multiply fractions, we take numerator times numerator, and denominator times denominator.

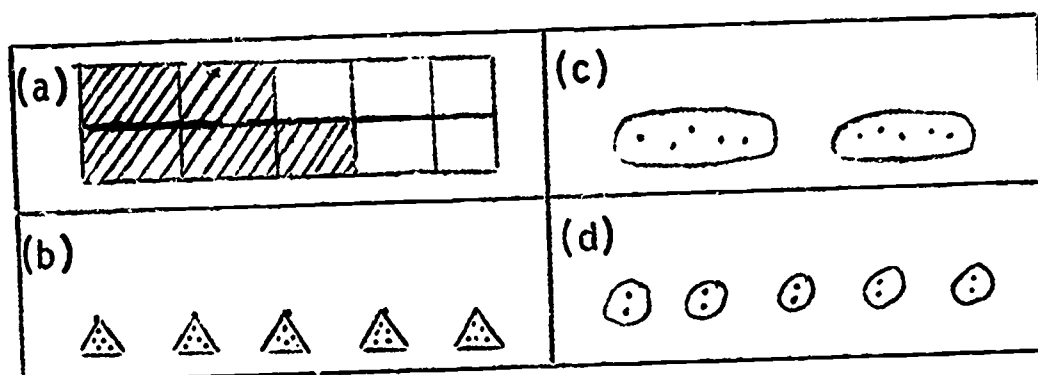
Example:

$$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}.$$

Go back to page 12 and try again.

You made the correct choice.

Now, which model shows $10 \div 5$?



- (a) Turn to page 41
- (b) Turn to page 23
- (c) Turn to page 55
- (d) Turn to page 60

Incorrect.

Does $\frac{1}{2} \times 10 = 10 \times 2$, and $\frac{1}{7} \times 14 = 14 \times 7$?

- (a) Yes Turn to page 68
- (b) No Turn to page 29

Incorrect.

$$9/10 \div 3/5 = \frac{9/10 \times 5/3}{1} = 45/30 = 3/2.$$

Now try this one.

$$4/7 \div 2/3 = ?$$

(a) $6/7$

Turn to page 13

(b) $8/21$

Turn to page 16

(c) $7/6$

Turn to page 90

You didn't multiply correctly.

What happened?

- (a) I made a careless mistake Go to page 10 and try again
- (b) I need a little help with multiplication of fractions
Go to Unit 6 and review multiplication
of fractions

O.K. You realized that

5 was not equal to 20
and 2 was not equal to 98.

Now, multiplying by $\frac{1}{4}$ is the same as _____
by 4.

- (a) dividing Turn to page 60
- (b) multiplying Turn to page 68

Incorrect.

The model you chose shows $\frac{1}{2}$ of 10. Now we know that $\frac{1}{2}$ of 10 is not equal to $\frac{1}{5}$ of 10, don't we?

Go back to page 15 and make another choice.

END

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Incorrect.

You didn't read page 90 very well. When we divide,
we multiply by the reciprocal.

The reciprocal of $\frac{5}{7}$ is:

- | | |
|-------------------|------------------|
| (a) $\frac{7}{5}$ | Turn to page 90 |
| (b) $\frac{5}{7}$ | Turn to page 78 |
| (c) I don't know | Turn to page 100 |

Incorrect.

What is the reciprocal of $1\frac{3}{8}$?

(a) $\frac{8}{11}$

Turn to page 90

(b) $\frac{11}{8}$

Turn to page 78

Incorrect.

Surely you remember that to multiply fractions, we take numerator times numerator, and denominator times denominator.

Example:

$$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}.$$

Go back to page 12 and try again.

Correct. So now we have $4 \div 2/3 = 6$. But you should notice that $4 \times 3/2$ also gives 6. Therefore, the procedure we are going to use for division is multiplication by the reciprocal of the divisor.

Example:

$$3 \div 1/6 = 3 \times 6/1$$

$$\text{and } 6 \div 3/2 = 6 \times 2/3$$

$$17 \quad 7/5 = 17 \times \underline{\quad ? \quad}.$$

- (a) $2/5$ Turn to page 31
- (b) $5/2$ Turn to page 43
- (c) I need help with reciprocals
Turn to page 100

Your answer was "divide by 2." Good, that is correct!

Dividing a number by 7 is the same as:

- (a) multiplying by 7 Turn to page 66
- (b) multiplying by $1/7$ Turn to page 42
- (c) dividing by $1/7$ Turn to page 26

Incorrect.

Notice that if you multiply the two numbers you picked together you will not get 1.

REMEMBER: The product of two reciprocals is always 1.

Are 3 and $\frac{1}{3}$ reciprocals of each other?

- | | |
|---------|-----------------|
| (a) Yes | Turn to page 47 |
| (b) No | Turn to page 53 |

Well, you're almost correct.

But you must reduce all answers to lowest form before you can consider yourself finished.

45/30 reduces to 3/2 because

$$45/30 = \frac{\cancel{15} \times 3}{\cancel{15} \times 2} = 3/2.$$

Reduce 20/28.

(a) 5/7

Turn to page 40

(b) 4/3

Turn to page 39

Incorrect.

You need some help reducing fractions. Go to page 20 of Unit 3 for some more work on reducing fractions.

Then return to page 18 of this Unit.

Correct!

Let's continue.

What is $\frac{(4/7)}{2}$?

(a) $8/7$

Turn to page 14

(b) $2/7$

Turn to page 50

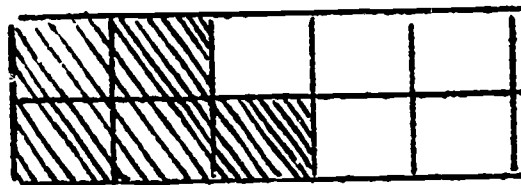
(c) $7/2$

Turn to page 54

Incorrect.

The model

shows $5/10$ or



5 divided by 10. You almost had it.

Go back to page 25 and try again.

Very good so far. But can you carry this reasoning one step farther? Let's see.

Multiplying a number by 2 is the same as:

- (a) dividing by 2 Turn to page 63
- (b) dividing by $1/2$ Turn to page 70
- (c) multiplying by $1/2$
 Turn to page 46

5/2 is correct!

What is the reciprocal of $2 \frac{1}{3}$?

- | | |
|-------------------------------------|-------------------------|
| (a) $\frac{7}{3}$ | Turn to page 78 |
| (b) $\frac{3}{7}$ | Turn to page 85 |
| (c) I don't know | Turn to page 100 |

Incorrect.

The symbol \div is read "divided by." The "divided by" number is always the divisor.

Example: $2/3 \div 6/7$ is read $2/3$ divided by $6/7$.

Therefore, $6/7$ is the divisor.

The divisor of $5/8 \div 7/4$ is:

(a) $7/4$

Turn to page 73

(b) $5/8$

Turn to page 83

Page 45

Your answer was 2 and $\frac{1}{2}$. Correct!

Consider the number 1. Is it a reciprocal of itself?

(a) Yes

Turn to page 47

(b) No

Turn to page 61

Page 46

You didn't read page 60 very well.

Go back and reread the material paying special attention to the underlined words. You should do Okay on your next selection.

Go back to page 60.

"Yes" is the correct answer.

Which of the following are reciprocals of each other?

- | | |
|-------------------------------------|-----------------|
| (a) $\frac{2}{5}$ and 5 | Turn to page 52 |
| (b) $\frac{2}{5}$ and 2 | Turn to page 79 |
| (c) $\frac{2}{5}$ and $\frac{5}{2}$ | Turn to page 97 |

You are having trouble with the basic operations of multiplication and division as applied to integers.

Go see your teacher and tell him your problem.
Then return to page 15 of this Unit.

Good!

Now this one:

The reciprocal of $1 \frac{5}{8}$ is:

(a) $\frac{8}{13}$

Turn to page 86

(b) $\frac{8}{5}$

Turn to page 52

Page 50

Your last answer was correct!

You are doing fine. Now get Booklet II of this Unit
and continue with page 101.

Page 51

You didn't read page 70 carefully enough.

Go back and reread page 70.

Incorrect.

REMEMBER: The product of two reciprocals is 1.

Let's look at this problem:

Is the product of $\frac{3}{2}$ and $\frac{2}{3}$ equal to 1?

- (a) Yes, and they're reciprocals Turn to page 97
- (b) No, and they're not reciprocals Turn to page 53
- (c) I don't know what the word
"product" means Turn to page 64

2

Page 53

You seem to be having trouble.

Ask your teacher for help and then return to page 100
of this Unit.

Incorrect!

We never invert the dividend. We always multiply by the reciprocal of the divisor.

$$4/7 \div 2 = 4/7 \times 1/2 = 4/14 = 2/7.$$

When we divide $2/3$ by 5, this is equal to:

- | | |
|----------------------|-----------------|
| (a) $3/2 \times 5$ | Turn to page 15 |
| (b) $2/3 \times 5$ | Turn to page 16 |
| (c) $2/3 \times 1/5$ | Turn to page 12 |

Incorrect.

The model you chose shows $10 \div 2$. Now we know that $10 \div 2$ does NOT equal $10 \div 5$, don't we?

(a) Yes

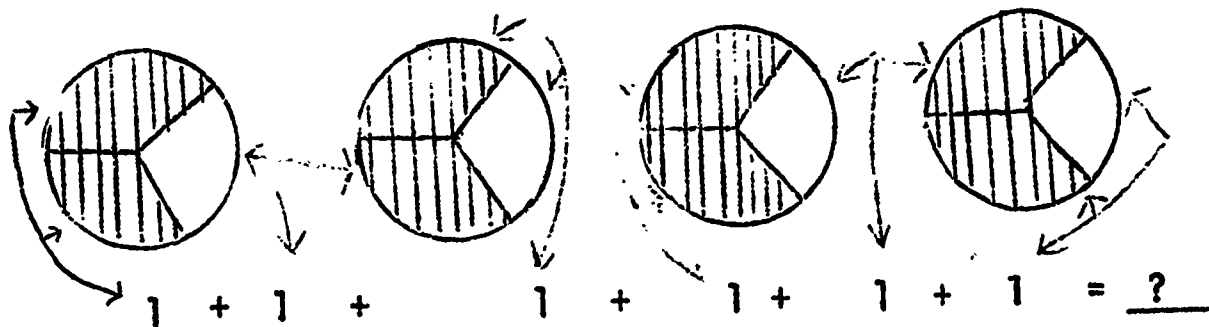
Then turn to page 25 and make another selection

(b) No

Go see your teacher and then return to page 15 of this Unit

Incorrect.

How many units of $\frac{2}{3}$'s? Well, let's count them.



- (a) There are 6 Turn to page 89
- (b) There are 4 Turn to page 58

Very good! 1 is a reciprocal of itself.

Are 2 and $\frac{1}{2}$ reciprocals of each other?

(a) Yes

Turn to page 47

(b) No

Turn to page 100

Page 58

Come now. I know you can count!

Let's try to get the right answer. Go to page 80
and try again.

O.K., you're on the right track now.

Multiplying a number by 2 is not equal to dividing the number by 2...

Then multiplying by 2 is equal to:

- (a) dividing by $1/2$ Turn to page 70
- (b) multiplying by $1/2$ Turn to page 46

Correct!

You should have observed that the same model that illustrates $\frac{1}{5}$ of 10 also illustrates $10 \div 5$.

IMPORTANT

Therefore, any model that shows division by 5 and multiplication by $\frac{1}{5}$ gives the same result.

Multiplying a number by $\frac{1}{2}$ gives the same result as:

- (a) multiplying by 2 Turn to page 26
- (b) dividing by 2 Turn to page 36
- (c) dividing by $\frac{1}{2}$ Turn to page 46

Page 61

Okay. That's better!

Does $2/3 \div 5/7 = 5/7 \div 2/3$?

(a) Yes

Turn to page 81

(b) No

Turn to page 76

You are moving along nicely now.

Now do this one.

What is the reciprocal of $3 \frac{1}{2}$?

- | | |
|-------------------|-----------------|
| (a) 7 | Turn to page 37 |
| (b) $\frac{2}{7}$ | Turn to page 86 |
| (c) $\frac{7}{2}$ | Turn to page 67 |

Incorrect.

Does dividing a number by two give you the same answer as multiplying by two?

In other words, does 12×2 give you the same answer as $12 \div 2$?

(a) Yes

Turn to page 48

(b) No

Turn to page 59

A product is the result of multiplying two or more numbers together.

Example: The product of 3×2 is 6.

Go back to page 52 and make another selection.

Good! We are doing very well now.

Here's another.

Does $\frac{3}{8} \div \frac{1}{2} = \frac{1}{2} \div \frac{3}{8}$?

(a) Yes

Turn to page 96

(b) No

Turn to page 77

Incorrect.

Let's see how it should be worked.

$$7/8 \div 3/4 = 7/8 \times 4/3 = \frac{7 \times \cancel{4}}{\cancel{4} \times 2 \times 3} = 7/6.$$

Try this one.

$$2/7 \div 1/3 = ?$$

(a) 6/7

Turn to page 95

(b) 7/6

Turn to page 71

(c) 2/21

Turn to page 90

Incorrect.

First, you write $3 \frac{1}{2}$ as a fraction and you get $\frac{7}{2}$.

Second, you invert to find its reciprocal.

What is the reciprocal of $6 \frac{3}{4}$?

- | | |
|--------------------|-----------------|
| (a) $\frac{4}{13}$ | Turn to page 52 |
| (b) $\frac{27}{4}$ | Turn to page 69 |
| (c) $\frac{4}{27}$ | Turn to page 49 |

You seem to be having trouble with equivalent relationships.

Go to Unit 3 and upon completion come back to page 15.

To invert means to turn upside down.

Examples:

When we invert $1/2$, we get $2/1$.

When we invert $3/13$, we get $13/3$.

When we invert x/y , we get y/x .

Now go to page 97 and make a selection.

Correct! Your answer was "divide by $1/2$."

You should have discovered that dividing by 2 is the same as multiplying by $1/2$.

Because 2 is the reciprocal of $1/2$, dividing any number by 2 is the same as multiplying by the reciprocal of 2.

$24 \div 6 = 24$ times the _____ of 6.

- (a) divisor Turn to page 51
- (b) reciprocal Turn to page 75
- (c) I don't know what a reciprocal is
 Turn to page 100

Sorry, but you missed it.

Let's look at the steps and see where you went wrong.

First, we have the question $2/7 \div 1/3 = ?$.

Second, division is the same as multiplication by the reciprocal $2/7 \times 3/1 = ?$.

Third, we multiply the two fractions and reduce if necessary $\frac{2 \times 3}{7 \times 1} = 6/7$.

Now you try it.

$$3/5 \div 2/3 = ?$$

(a) 10/9

Turn to page 90

(b) 15/6

Turn to page 82

(c) 9/10

Turn to page 95



Okay, so far!

Since integers can be written as fractions, (example:
3 can be written as $3/1$), we find the reciprocal of an
integer by writing it as a fraction and then inverting
it.

The reciprocal of 5 is:

(a) $1/5$

Turn to page 62

(b) $5/1$

Turn to page 37

(c) Help!

Turn to page 100

Correct!

$$5/7 \div 3/2 = ?$$

(a) $5/7 \times 2/3$

Turn to page 65

(b) $7/5 \times 3/2$

Turn to page 44

(c) $5/7 \times 3/2$

Turn to page 32

Incorrect.

Isn't 1×1 equal to 1? Then 1 is a reciprocal of itself.

Turn to page 47.

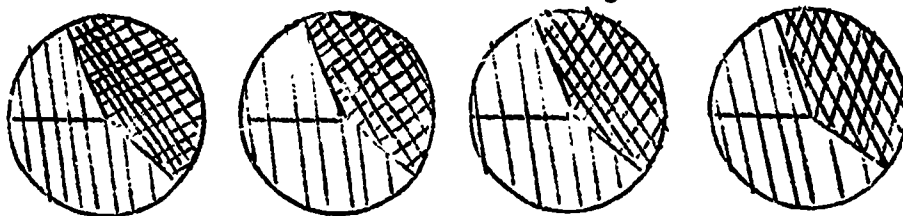
Let's discuss division of fractions in detail now.

Consider the problem $4 \div \frac{1}{3}$. This can be stated as

"How many $\frac{1}{3}$'s are there in 4?" We can illustrate

this by means of a model. Look at the figure to

your right.



We can see 3 such units of " $\frac{1}{3}$ " in each circle.

Four circles times 3 gives us 12 such units in all.

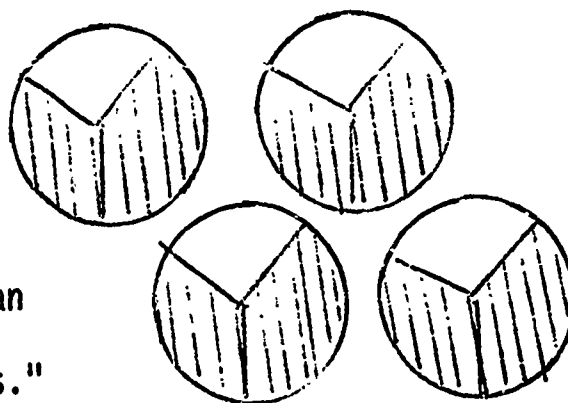
Hence, $4 \div \frac{1}{3} = 12$.

Consider $4 \div \frac{2}{3}$. Stating

it in the form of "How many

$\frac{2}{3}$'s are there in 4?", we can

count _____ units of " $\frac{2}{3}$'s."



(a) 6

Turn to page 35

(b) 4

Turn to page 80

(c) 12

Turn to page 56

No is the correct answer!

Here is another problem.

Does $\frac{2}{3} \div \frac{3}{5} = \frac{3}{5} \div \frac{3}{2}$?

(a) Yes

Turn to page 90

(b) No

Turn to page 84

"No" is the correct answer.

Now let's work the problems completely.

$$7/8 \div 3/4 = ?$$

- (a) $6/7$ Turn to page 66
- (b) $7/6$ Turn to page 88
- (c) Neither answer is correct
Turn to page 92

Page 78

You need more work with reciprocals.

Go to page 100.

Incorrect.

Remember: The product of two reciprocals is 1.

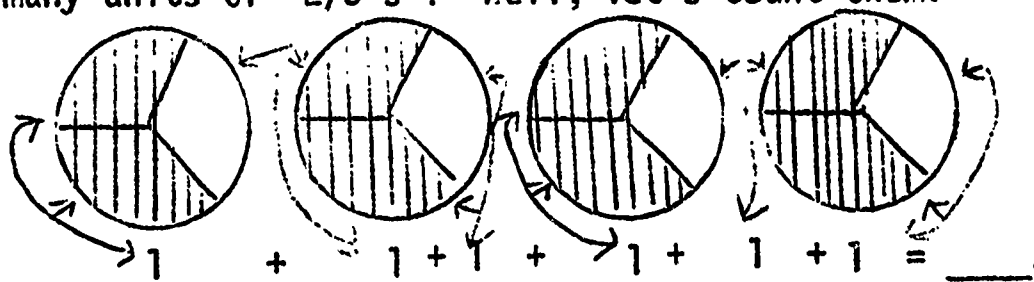
Let's look at this problem:

Is the product of $3/2$ and $2/3$ equal to 1?

- (a) Yes, and they're reciprocals Turn to page 97
- (b) No, and they're not reciprocals Turn to page 53
- (c) I don't know what the word "product" means
 Turn to page 64

Incorrect.

How many units of "2/3's"? Well, let's count them.



(a) There are 6 Turn to page 89

(b) There are 4 Turn to page 58

Ooops! You made a mistake.

Now you know that:

$$2/3 \div 5/7 = 2/3 \times 7/5$$

and

$$5/7 \div 2/3 = 5/7 \times 3/2.$$

Now, does $2/3 \times 7/5 = 5/7 \times 3/2$?

- | | |
|---------|-----------------|
| (a) Yes | Turn to page 68 |
| (b) No | Turn to page 76 |

Incorrect.

When we divide, we find ONLY the reciprocal of the divisor.

In the problem $3/2 \div 4/7$, the reciprocal of the divisor is:

(a) $7/4$

Turn to page 73

(b) $4/7$

Turn to page 100

(c) $2/3$

Turn to page 44

8

Page 83

Come now. You're not trying.

Go back and reread page 44 and try again.

"No" is correct.

Let's work a few out completely.

$$4/5 \div 3/10 = ?$$

(a) $25/6$

Turn to page 82

(b) $3/8$

Turn to page 91

(c) $8/3$

Turn to page 87

Your answer was $3/7$. Very good! You seem to be getting it.

Let's continue.

$14 \div 2 \frac{1}{3}$ is equal to 14 times the reciprocal of $2 \frac{1}{3}$. Writing this, we should have:

$$\boxed{14 \div 2 \frac{1}{3} = 14 \times \frac{3}{7}} \text{ , as } \frac{3}{7} \text{ is the } \underline{\text{reciprocal}}$$

of $2 \frac{1}{3}$.

Now you try one.

$$8 \div 1 \frac{3}{8} = 8 \times \underline{\quad ? \quad}.$$

(a) $1 \frac{3}{8}$

Turn to page 51

(b) $11/8$

Turn to page 33

(c) $8/11$

Turn to page 90

Very good! Here is the last thing you need to know.

That is--The number 0 (zero) has no reciprocal.

In other words, what number times zero gives "1"?

There isn't any, is there?

Now go to page 90 and continue your Unit on division of fractions.

$8/3$ was the correct answer.

Now do this one.

$$7/8 \div 3/4 = ?$$

(a) $6/7$

Turn to page 91

(b) $7/6$

Turn to page 18

(c) $32/21$

Turn to page 82

Page 88

Okay, so far. $7/6$ is correct!

Let's try again.

$$4/5 \div 3/10 = ?$$

(a) $8/3$

Turn to page 18

(b) $3/8$

Turn to page 90

(c) $25/6$

Turn to page 82

Good!

Now go back to page 75 and reread the material carefully. Then rework the problem on that page.

Now let's look at division of fractions through the use of a general rule.

The method or rule that we will use to explain division of fractions is:

Division of a number is equal to the number multiplied by the reciprocal of the divisor.

Example: $2/3 \div 1/4 = 2/3 \times 4/1 = 8/3$.

$$2/3 \div 5/7 = ?$$

- | | |
|----------------------|-----------------|
| (a) $2/3 \times 5/7$ | Turn to page 32 |
| (b) $2/3 \times 7/5$ | Turn to page 65 |
| (c) $3/2 \times 5/7$ | Turn to page 82 |
| (d) $3/2 \times 7/5$ | Turn to page 82 |

Incorrect.

Let's look at an example to see how it's done.

Example: $2/3 \div 4/5 = ?$

First, we change the division problem to a multiplication problem by multiplying $2/3$ by $5/4$ which is the reciprocal of the divisor.

Hence, $2/3 \div 4/5 = 2/3 \times 5/4$.

Now multiply and reduce. $2/3 \times 5/4 = \frac{\cancel{2} \times 5}{3 \times \cancel{2} \times 2} = 5/6$.

Try this one.

$5/6 \div 1/3 = ?$

(a) $5/2$

Turn to page 99

(b) $18/5$

Turn to page 82

(c) $2/5$

Turn to page 94

Incorrect.

Let's look at the solution to this problem.

$$7/8 \div 3/4 = 7/8 \times 4/3 = 28/24.$$

However, 28/24 will reduce to:

- | | |
|-----------|-----------------|
| (a) 6/7 | Turn to page 39 |
| (b) 7/6 | Turn to page 88 |
| (c) 14/12 | Turn to page 33 |

Very good! That's correct.

Work this problem.

$$5/6 \div 2/3 = ?$$

(a) $9/5$

Turn to page 82

(b) $5/4$

Turn to page 18

(c) $4/5$

Turn to page 98

Wrong again. Let's look at how it is worked.

$$5/6 \div 1/3 = 5/6 \times 3/1.$$

Now, $5/6 \times 3/1 =$

$$\frac{5 \times 3}{2 \times 1} = 5/2.$$

Try this one.

$$1/4 \div 2/5 = ?$$

(a) $8/5$

No. You multiplied by the reciprocal of the dividend. Turn to page 75 for more work.

(b) 10

Incorrect. Turn to page 15 and study division in detail.

(c) Neither of the above

Correct. Now that you have the idea, let's go page 90 and work a little more carefully.

Page 95

Good! Your answer was correct.

Do this problem:

$$3/4 \div 1/2 = ?$$

(a) $3/8$

Turn to page 15

(b) $2/3$

Turn to page 91

(c) $3/2$

Turn to page 93

Page 96

Doggone it! I thought you had it there for a minute.

I'm sure you remember that $3/8 \div 1/2 = 3/8 \times 2/1$
and that $1/2 \div 3/8 = 1/2 \times 8/3$.

Now I ask you, "Does $3/8 \times 2/1 = 1/2 \times 8/3$?"

- | | |
|---------|-----------------|
| (a) Yes | Turn to page 68 |
| (b) No | Turn to page 61 |

Correct! Let's continue.

You have probably noticed that to find a reciprocal of a fraction, you just invert it.

The reciprocal of $\frac{3}{8}$ is:

- (a) $\frac{3}{8}$ Turn to page 52
- (b) $\frac{8}{3}$ Turn to page 72
- (c) 8 Turn to page 37
- (d) I don't know what the word "invert" means
Turn to page 69

Shucks! You were almost through.

You're supposed to multiply by the reciprocal of the divisor, not the reciprocal of the dividend.

In other words, you should have

$$\boxed{5/6 \div 2/3 = 5/6 \times 3/2} \quad \text{and } \underline{\underline{\text{not}}}$$

$$5/6 \div 2/3 = 6/5 \times 2/3.$$

Go to page 91 and go from there.

Page 99

Your answer was $5/2$. Good, that's correct!

Try this one.

$$5/12 \div 1/4 = ?$$

(a) $5/3$

(b) $3/5$

(c) $5/48$

Turn to page 93

Turn to page 75

Turn to page 15

So you're having trouble with reciprocals. Well, let's look at the definition and a few examples. Then I think you'll be okay.

Reciprocals Defined

When two numbers have a product of 1, each of the numbers is called the "reciprocal" of the other.

Examples: $(1/4)(4) = 1$ $(2/3)(3/2) = 1$
 $(5/2)(2/5) = 1$

Now, $1/4$ and 4 , $2/3$ and $3/2$, and $5/2$ and $2/5$ are reciprocals of each other.

Which of the following numbers are reciprocals of each other?

- | | |
|---------------------|-----------------|
| (a) $6/5$ and $1/6$ | Turn to page 37 |
| (b) 1 and 1 | Turn to page 57 |
| (c) $1/4$ and $1/4$ | Turn to page 37 |
| (d) 2 and $1/2$ | Turn to page 45 |